A NOTE ON INVARIANCE IN THE BEHAVIORAL APPROACH*

PAULA ROCHA† AND RICARDO PEREIRA‡

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EXTENDED ABSTRACT. This talk is a first step towards the extension of the basic notions of invariance, controlled and conditioned invariance from the geometric theory (see [1]) to the case of behavioral systems.

We consider smooth linear differential behaviors $\mathcal{B} = \{w \in \mathcal{W} : \ R(\sigma)w = 0\} =: \ker R(\sigma)$, where $\mathcal{W} := \mathcal{C}^\infty(\mathbb{R}, \mathbb{R}^{\mathtt{w}})$, σ denotes the differential operator $\frac{d}{dt}$, and $R(s) \in \mathbb{R}^{g\times \mathtt{w}}[s]$ is a polynomial matrix, called *representation* of $\mathcal{B}$.

Given two behaviors $\mathcal{B}$ and $\mathcal{V}$, we say that $\mathcal{V}$ is $\mathcal{B}$-*invariant* if the following condition holds:

$$[w \in \mathcal{B},\ t_0 \in \mathbb{R},\ w_{|(-\infty,t_0]} \in \mathcal{V}_{|(-\infty,t_0]}] \Rightarrow [w \in \mathcal{V}],$$

where, as usual, $w_{|(-\infty,t_0]}$ denotes the restriction of w to the interval $(-\infty, t_0]$ and $\mathcal{V}_{|(-\infty,t_0]}$ is the set of restrictions of the trajectories $v \in \mathcal{V}$ to the that same interval.

In order to define controlled invariance, recall that, in the behavioral framework, controlling a behavior $\mathcal{B}$ consists in finding a controller behavior $\mathcal{K} = \ker K(\sigma)$ such that its interconnection with $\mathcal{B} = \ker R(\sigma)$ yields a desired behavior (control objective) $\mathcal{B}^d$, i.e., such that $\mathcal{K} \cap \mathcal{B} = \mathcal{B}^d$. Of particular interest are regular controllers, which are the ones that impose restrictions that do not overlap with the ones corresponding to the original behavior, [4].

Given a behavior $\mathcal{B}$, we say that the behavior $\mathcal{V}$ is *controlled-invariant* (with respect to $\mathcal{B}$) if there exists a regular controller $\mathcal{K}$, for $\mathcal{B}$, such that $\mathcal{V}$ is $\big(\mathcal{B} \cap \mathcal{K}\big)$-invariant. This means that it is possible to control the behavior $\mathcal{B}$ in such a way that trajectories of the controlled behavior "starting in $\mathcal{V}$" do not leave $\mathcal{V}$.

In order to define conditioned invariance, consider a behavior $\mathcal{B} \subset \mathcal{W}_1 \times \mathcal{W}_2$ whose variable w is partitioned into two (vector valued) variables w_1 and w_2 in such a way that w_1 represents the system attributes that can be directly measured and w_2 corresponds to the remaining (say, internal) attributes. We say that a behavior

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†Department of Electrical and Computer Engineering, Faculty of Engineering, University of Porto, Rua Dr. Roberto Frias, 4200-465, Porto, Portugal (`mprocha@fe.up.pt`)

‡Department of Mathematics, University of Aveiro, Campus de Santiago, 3810-193, Aveiro, Portugal (`ricardopereira@ua.pt`)

$\mathcal{V} \subset \mathcal{W}_2$ is *conditioned-invariant* (with respect to $\mathcal{B}$) if there exists a third behavior $\hat{\mathcal{B}} \subset \mathcal{W}_1 \times \mathcal{W}_2$, with variable $(w_1, \hat{w}_2)$, such that the following conditions are satisfied:

$$[(w_1, w_2) \in \mathcal{B}] \Rightarrow [\exists \hat{w}_2 : (w_1, \hat{w}_2) \in \hat{\mathcal{B}}],$$

$$[(w_1, w_2) \in \mathcal{B} \wedge (w_1, \hat{w}_2) \in \hat{\mathcal{B}} \wedge \left(w_2 - \hat{w}_2\right)_{|(-\infty,0]} \in \mathcal{V}] \Rightarrow [w_2 - \hat{w}_2 \in \mathcal{V}].$$

The first condition means that $\hat{\mathcal{B}}$ imposes no extra restrictions on w_1 beyond the ones already imposed by $\mathcal{B}$, i.e., $\hat{\mathcal{B}}$ is *nonintrusive* (see [3],[2]). The second condition means that $\hat{\mathcal{B}}$ is an observer for $\mathcal{B}$ ([3]) "modulo $\mathcal{V}$".

The purpose of this talk is to analyze the proposed definitions in detail, to characterize them in terms of the representations of the involved behaviors, and to compare them with the notions defined for state space systems. The existence and characterization of the largest controlled-invariant sub-behavior of a given behavior $\mathcal{V}$, as well as of the smallest conditioned-invariant behavior containing a given behavior $\mathcal{V}$, will be investigated.

REFERENCES

[1] G. Basile, and G. Marro, Controlled and conditioned invariant subspaces in linear system theory, *Journal of Optimization Theory and Applications*, Vol. 3, N. 5, 1969, pp. 306 - 315.
[2] J. Trumpf, H.L. Trentelman, and J.C. Willems, An internal model principle for observers, Proceedings of the 50th IEEE Conference on Decision and Control and European Control Conference, 2011, pp. 3992 - 3999.
[3] M.E. Valcher and J. C. Willems, Observer Synthesis in the Behavioral Approach, IEEE Transactions on Automatic Control, Vol. 44, N.12, 1999, pp. 2297 - 2307.
[4] J.C. Willems, On interconnections, control, and feedback, *IEEE Transactions on Automatic Control*, Vol.42, 1997, pp. 326 - 339.