GLOBAL BIFURCATIONS OF CONTROL SETS AND RANDOM DYNAMICS*

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The global controllability properties of nonlinear control systems are closely related to the analysis of associated random dynamical systems. Under appropriate ergodicity and hypoellipticity assumptions, one formally replaces the deterministic control term by a random noise. Then the invariant subsets of complete controllability, called the invariant control sets, determine the supports of the invariant measures. If, under small perturbations, invariance is lost, one expects that the perturbed system still shows similar, although transient behavior. In particular, exit from the formerly invariant subset occurs only on a much longer time scale. This is well documented in numerical studies, and a standard method for the analysis is the theory of large deviations.

This translation mechanism applies to random differential equations as well as to random diffeomorphisms with special features in the latter context. For example, the invariant control sets need not be connected. In the continuous time setting, these results are due to Stroock and Varadhan, Kunita, Arnold and Kliemann. On the other hand, the transient behavior of random systems can be described by conditionally invariant measures, i.e., measures which are conditioned with respect to a transient set. The transient behavior is completely determined by the supports of the conditionally invariant measures. Hence it is of interest to determine these supports.

This paper restricts attention to discrete-time control systems and the associated random diffeomorphisms. Assumptions are made, which guarantee that the invariant measures have densities with respect to Lebesgue measure. Then a study of the associated conditioned Perron-Frobenius operator, its spectrum and its eigenvectors shows that also the associated conditionally invariant measures have densities with respect to Lebesgue measure and that the densities depend continuously on parameters. For the associated (deterministic) control system one can perform an analogous perturbation analysis. Here the invariant control sets turn into control sets which are no more invariant.

One can show that invariant control sets lose their invariance only if they change discontinuously in the Hausdorff metric. However, if one changes the analysis by conditioning them to a given subset W of the state space, they turn into conditionally invariant control sets which may change continuously. Relatively invariant control sets have several properties which are analogous to those of invariant control sets. For example, under weak assumptions, existence is guaranteed and they are closed relative to W (instead of closed), they are pairwise disjoint and their number is finite.

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These similarities may lead us to the conjecture that for conditionally invariant measures relatively invariant control sets play a role, which is analogous to the role of invariant control sets for invariant measures. Using a perturbation approach, this paper confirms this conjecture for a certain class of random diffeomorphisms. If one is far from the invariance situation, the conjecture remains open.

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