PORT HAMILTONIAN MODELING OF POWER NETWORKS
F. SHAIK *, D. ZONETTI †, R. ORTEGA †, J.M.A. SCHERPEN*, AND A.J. VAN DER SCHAF'T *

Key words. power networks, port-Hamiltonian systems, bond-graphs, structure preserving models, graph

AMS subject classifications. 93-06, 93Axx, 93Bxx

1. Introduction. To study the power system stability, in the past, power engineers have developed simplified, reduced order, models that neglect some fast transients and losses. In particular, it is assumed that the electrical magnitudes can be represented via (first harmonic) phasors, and the generator dynamics is reduced to a second or third order model. Unfortunately, these reductions destroy the physical structure of the system, leading to some approximate rationalizations of the new quantities, e.g., the concept of “voltage behind the reactance”, and an awkward interpretation of basic physical concepts like energy and dissipation, which are introduced only for mathematical convenience [1].

To deal with nonlinear, multi-domain, systems we have witnessed in the last few years an increasing interest in energy–based modeling, analysis and controller design techniques. In particular, the use of port–Hamiltonian (pH) systems has proven highly successful in many applications, see [2, 7] and references therein.

A port-Hamiltonian system (in input-state-output form) is given by
\[
\dot{x} = [\beta(x) - \mathcal{R}(x)] \nabla H(x) + g(x)u,
\]
\[y = g^T(x) \nabla H(x)
\] (1.1)

where \(\beta^t(x) = -\beta(x),\ \mathcal{R}^t(x) = \mathcal{R}(x) \geq 0,\) and \(\nabla H(x)\) is the vector of partial derivatives of the Hamiltonian \(H(x)\) with respect to the state \(x\).

In this paper starting from first principles, without afore mentioned simplifying assumptions, a full, nonlinear model for the power network in port-Hamiltonian framework is derived. For this we use the modularity approach i.e., we first derive the models of individual components in power network as port-Hamiltonian systems [2] and then combine all the component models using power-preserving interconnections to give a global port-Hamiltonian model. In this way we obtain a structure-preserving disaggregated model that basically preserves the original topology of the network, which will subsequently pave the way for energy based analysis for the stability studies.

2. Port–Hamiltonian Models of Generators, Lines and Loads. A power network can be viewed as a graph where generators, transmission lines and loads correspond to edges and the buses correspond to nodes. We call a bus generator bus if a generator is connected to it and we call a bus load bus when a load is connected to it. We call a bus a reference bus when all the voltages of the buses in the network are measured with respect to it. Usually a reference bus is at ground potential.

*Faculty of Mathematics and Natural Sciences, University of Groningen, P. O Box 800, 9700 AV Groningen, The Netherlands f.shaik@rug.nl, A.J.van.der.Schaft@math.rug.nl, j.m.a.scherpen@rug.nl
†Laboratoire des Signaux et Systèmes CNRS-Supelec, Plateau de Moulon, Supelec, 91192, Gif-sur-Yvette, France daniele.zonetti@gmail.com, romeo.ortega@lss.supelec.fr
For simplicity in this work we assume that there are no buses other than reference bus which is neither a generator bus nor a load bus. Let there are \( g \) generator buses and \( l \) load buses and one reference bus. Then the total number of buses (nodes) in the network is \( n = g + l + 1 \). Without loss generality we assume that the nodes 1, \ldots, g are generator nodes and \( g + 1, \ldots, g + l \) are load nodes and node \( n \) is the reference node.

There is a generator edge between every generator node and the reference node and there is a load edge between load node and the reference node. Therefore there are in total \( g \) generator and \( l \) load edges. Let there are \( T \) number of transmission lines connecting different buses. In this work we use the general lossy Π-model \([1]\) to represent the transmission line as shown in Figure 2.1.

From Figure 2.1 we can see that there are three edges for each transmission line: one edge corresponding to \( R - L \) series circuit between the same nodes as of transmission line and two capacitor edges between these nodes and the reference node. Therefore for \( T \) transmission lines there are \( T \) \( R - L \) series circuit edges and \( 2T \) capacitor edges. Hence there are in total \( m = g + l + 3T \) number of edges. Without loss of generality assume that edges 1, \ldots, \( g \) are edges corresponding to generators, \( g + 1, \ldots, g + l \) are edges corresponding to loads, \( g + 1 + l, \ldots, g + 1 + T \) are edges corresponding to \( R - L \) series circuits of transmission lines, and \( g + 1 + T, +1, \ldots, m \) are edges corresponding to capacitor edges.

We note that contrary to \([5]\) in the power network graph given in this work edges (only) have dynamics. In the remainder of this section we give detailed models for these different edges.

2.1. Edges corresponding to generators \((e = 1, \ldots, g)\). Using bond graph modeling \([2]\) each synchronous machine is described by the following equations (for the details see \([1],[6]\))

\[
\begin{bmatrix}
\Psi_{es} \\
\Psi_{er} \\
p_e \\
\theta_e
\end{bmatrix}
= 
\begin{bmatrix}
-R_{es} & 0 & 0 & 0 \\
0 & -R_{er} & 0 & 0 \\
0 & 0 & -d_e & -1 \\
0 & 0 & 1 & 0
\end{bmatrix}
\nabla H_e
+ 
\begin{bmatrix}
I_3 \\
0_{3 \times 1} \\
0_{3 \times 1} \\
0_{2 \times 1}
\end{bmatrix}
\begin{bmatrix}
V_e \\
E_{ef} \\
T_{em}
\end{bmatrix}
\]

\[
H_e = \frac{1}{2} \left[ \begin{array}{cc}
\Psi_{es} & \Psi_{er}
\end{array} \right] L^{-1}(\theta_e) \left[ \begin{array}{cc}
\Psi_{es} & \Psi_{er}
\end{array} \right]^T + \frac{1}{2J_e} p_e^2.
\]

(2.1)
where $\Psi_e, V_e, I_e, R_e$ are 3 phase stator flux linkages, voltages, currents and resistances respectively, $\Psi_r, E_{ef}, I_{ef}, R_{er}$ are rotor flux linkages, applied field voltage, field current, and rotor circuit resistances respectively, $\theta_e, \omega_e$ and $p_e$ are angular displacement, angular velocity and momentum of rotor respectively and finally $T_{em}$ represents the mechanical torque applied.

2.2. Edges corresponding to loads ($e = g + 1, \ldots, g + 1$). Modeling of nonlinear loads in a power network is an unsolved problem. Therefore, for the sake of simplicity in this paper we represent them by dissipative elements characterized by the following generalized relations:

$$r_e(u_{Le}, y_{Le}) = 0$$

(2.2)

where $u_{Li}$ and $y_{Li}$ are the voltage across and current flowing in the load $i$, respectively.

2.3. Edges corresponding to $R - L$ series circuit of transmission lines ($e = g + 1 + T, \ldots, g + 1 + T$). Edges corresponding to $R - L$ series circuit of transmission line have the following dynamics

$$\dot{\Psi}_e = -R_e \nabla_{\Psi_e} H_e(\Psi_e) + V_e$$

$$I_e = \nabla_{\Psi_e} H_e(\Psi_e)$$

$$H_e(\Psi_e) = \frac{1}{2} \Psi_e^T L_e^{-1} \Psi_e$$

(2.3)

where $V_e, I_e$ and $\Psi_e$ are voltage across, current through and flux linkages in inductors of 3-phase $R - L$ series circuit respectively, $R_e$ and $L_e$ are resistances and inductances of 3-phase $R - L$ series circuit.

2.4. Edges corresponding to capacitors of transmission line ($e = g + 1 + T + 1, \ldots, m$). As there might be more than one transmission lines connected to each bus, from the Π model of transmission lines it is easy to see that we might encounter two or more capacitors in parallel at a given bus. For simplicity we replace all parallel capacitors at a given bus by a single capacitor with an equivalent capacitor. The dynamics of capacitor edges are given by

$$\dot{Q}_e = I_e$$

$$V_e = \nabla_{Q_e} H_e(Q_e)$$

$$H_e(Q_e) = \frac{1}{2} Q_e^T C_e^{-1} Q_e$$

(2.4)

where $C_e, V_e, I_e$ and $Q_e$ are 3 phase capacitance, voltage across, current through and charges in the capacitors respectively. Capacitor edges are connected to either generator buses or load buses. Let there are $T_{eg}$ number of capacitors connected to generator buses and $T_{el} = 2T - T_{eg}$ number of capacitors connected to load buses. Without loss of generality assume that edges $e = g + 1 + T + 1, \ldots, g + 1 + T + T_{eg}$ correspond to capacitors connected to generator buses and $e = g + 1 + T + T_{eg} + 1, \ldots, m$ correspond to capacitors connected to load buses.

2.5. Interconnection laws. If an oriented incidence matrix of the graph is given by

$$M = \begin{bmatrix}
I_{3g} & 0_{3g \times 31} & M_1 & I_{3g} & 0_{3g \times 31} \\
0_{31 \times 3g} & I_{31} & M_2 & 0_{31 \times 3g} & I_{31} \\
-1_{3g} & -1_{31} & 0_{1 \times 37} & -1_{3g} & -1_{31}
\end{bmatrix}$$

(2.5)
then the interconnection laws (KCL and KVL) of the power network are given by [4]
\[
\begin{align*}
M_1 \varphi &= 0 \\
M^T \varphi &= \varphi_e
\end{align*}
\]  \hspace{1cm} (2.6)

where \( \varphi_e = \text{col}(I_1, \ldots, I_n) \), \( \varphi' = \text{col}(V_1, \ldots, V_n) \) and \( \varphi' = \text{col}(v_1, \ldots, v_n) \) are vectors of 3 phase edge currents, edge voltages and node voltages, respectively.

In (2.5), the matrix \( M' := \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} \) represent an oriented incidence matrix of the sub-graph obtained by eliminating the reference node and edges incident at this node. The incidence matrix \( M' \) captures the information about the interconnection structure of generators and loads.

3. Global model. By combining all the individual models of the power network components given in section 2 and eliminating network constraints given by (2.6) we obtain a global port–Hamiltonian power network model as

\[
\begin{bmatrix}
\Psi_{SG} \\
\Psi_T \\
Q_{GT} \\
Q_{LT} \\
\Psi_{RG} \\
\Psi_G \\
\Theta_G \\
\omega_G \\
V_L
\end{bmatrix}
= \begin{bmatrix}
-R_{SG} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -R_T & M_1 & M_2 & 0 & 0 & 0 & 0 & 0 \\
-1 & -M_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -R_{cG} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -D & -I & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \nabla H + \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
E_f & 0 & 0 & 0 & 0 & 0 & 0 & 0 & T_m \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]  \hspace{1cm} (3.1)

where \( H = \Sigma_{e=1} H_e \). In above equations \( \Psi_{SG} = \text{col}(\Psi_{1x}, \ldots, \Psi_{gx}) \), \( \Psi_{RG} = \text{col}(\Psi_{1r}, \ldots, \Psi_{gr}) \), \( \Psi_T = \text{col}(\Psi_{g+1+1}, \ldots, \Psi_{g+1+r}) \), \( Q_{GT} = \text{col}(Q_{g+1+r+1}, \ldots, Q_{g+1+r+2}) \), \( Q_{LT} = \text{col}(Q_{g+1+r+1}, \ldots, Q_{g}) \), \( \bar{E}_f = \text{col}(\bar{E}_{1f}, \ldots, \bar{E}_{1g}) \), \( \bar{I}_f = \text{col}(\bar{I}_{1f}, \ldots, \bar{I}_{1g}) \),

\[
\begin{align*}
\omega_G & = \text{col}(\omega_1, \ldots, \omega_2) \\
\Theta_G & = \text{col}(\theta_1, \ldots, \theta_2) \\
V_L & = \text{col}(V_{g+1}, \ldots, V_{g+1}) \\
I_L & = \text{col}(I_{g+1}, \ldots, I_{g+1})
\end{align*}
\]

REFERENCES