SAFETY ANALYSIS UNDER PROBABILISTIC TARGET CONSTRAINTS

M. ASSELLAOU*, O. BOKANOWSKI†, A. DESILLES‡, AND H. ZIDANI§

Key words. stochastic control, probabilistic target, safety region, level sets, Semi-Lagrangian scheme

AMS subject classifications. 93E20, 49L25, 49L20

This work focuses on a safety analysis problem under probabilistic target constraints. Given a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and filtration $(\mathcal{F}_t)_{t\in[0,T]}$, we consider a controlled stochastic differential equation $X_s = X^u_{t,x}(s)$ in $\mathbb{R}^d$:

$$dX(s) = b(s, X(s), u(s))ds + \sigma(s, X(s), u(s))dW(s), \qquad s \in [t, T],$$
$$X(t) = x,$$

where $x \in \mathbb{R}^d$, $T > 0$, u belongs to the control set $\mathcal{U}$ (the set of progressively measurable functions taking values in a given compact set U), $W(s)$ is an m-dimensional Brownian motion, and $\sigma : [0, T] \times \mathbb{R}^d \times U \to \mathbb{R}^{d\times m}$ and $b : [0, T] \times \mathbb{R}^d \times U \to \mathbb{R}^d$ are given functions.

Given a non empty closed set $\mathcal{C}$ of $\mathbb{R}^d$, the problem consists in finding the set Ω^ρ_t of all points x such that there exists some control law u and an associated process $X^u_{t,x}$ that belongs to $\mathcal{C}$ at time T with probability larger than ρ:

$$\mathbb{P}[X^u_{t,x}(T) \in \mathcal{C}] \geq \rho,$$

where $t > 0$ and $\rho \in [0, 1]$ corresponds to a given probability of success we want to achieve.

The main motivation of this study is the collision avoidance problem under uncertainties on the dynamics. Usually this class of problems is studied in the two-player games setting, where the (deterministic) uncertainties are modelized as a second player and the safety regions are determined by considering the "worst-case" on the uncertainties. Here, our aim is to decompose the domain into different safety regions corresponding to different levels of risk.

In order to characterize Ω^ρ_t, one can extend the level-set approach and consider a control problem with proabilistic state constraints. Such problems have been recently studied, see for instance [1].

Here, we investigate a simpler idea, and consider the control problem

$$v(t, x) := ess \sup_{u\in\mathcal{U}} \mathbb{P}\big[X^u_{t,x}(T) \in \mathcal{C}\big].$$

*Dép. math. appliquées, ENSTA ParisTech, 32 Bd Victor 75739 Paris Cx 15. Email: mohamed.asselaou@ensta-paristech.fr

†Laboratoire Jacques-Louis Lions, Université Pierre et Marie Curie 75252 Paris Cx 05 France & UFR de Mathématiques, Site Chevaleret, Université Paris-Diderot, 75205 Paris Cedex France & Projet Commands INRIA Saclay - ENSTA ParisTech. Email: bokanowski@ljll.ann.jussieu.fr

‡Dép. math. appliquées, ENSTA ParisTech, 32 Bd Victor 75739 Paris Cx 15. Email: anna.desilles@ensta-paristech.fr

§Projet Commands, ENSTA ParisTech & INRIA Saclay, 32 Bd Victor 75739 Paris Cx 15. Email: Hasnaa.Zidani@ensta-paristech.fr

With this single control problem, each level set of $v(t, \cdot)$ will correspond to a safety region Ω_t^ρ with an adequate risk level ρ. Moreover, the value v can be characterized in terms of a second order Hamilton-Jacobi-Bellman equation. An approximation procedure will be proposed, using a new semi-Lagrangian scheme, based on ideas of [2, 3].

This study will be illustrated on a simple model of collision avoidance involving an UAV (unmanned aircraft vehicule) and an intruder aircraft (see for instance [4]).

Acknowlegments. This works is partially supported by the "DGA - Research department in the French Defence Ministry", under the Grant ENSTA-DGA number 0660037, and partially by the EU under the 7th Framework Programme Marie Curie Initial Training Network FP7-PEOPLE-2010-ITN, SADCO project, GA number 264735-SADCO.

REFERENCES

[1] B. Bouchard, R. Elie, N. Touzi, *Stochastic target problems with controlled loss,* SIAM J. Control Optim. (2009), Vol. 48, No. 5, pp. 3123–3150

[2] K. Debrabant, E. R. Jakobsen, *Semi-Lagrangian schemes for linear and fully non-linear diffusion equations.* To appear in Math. Comp.

[3] O. Bokanowski, G. Simarmata, *Semi-Lagrangian discontinuous Galerkin schemes for first and second order partial differential equations*, Preprint.

[4] E. Cruck, A. Désilles, H. Zidani, *Collision analysis for an UAV*, To appear in AIAA, J. of Guidance and Control.