SYNCHRONIZATION ANALYSIS OF MULTI-AGENT SYSTEMS WITH SWITCHING TOPOLOGIES

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EXTENDED ABSTRACT. The distributed control of multi-agent systems has gained lots of attention during the last decade. In particular, consensus problem has been widely investigated for networks of agents. Consensus roughly means that the agents of a network reach an agreement on the state components' values. The pioneering works in this regard have been carried out for the case where the agents have simple dynamics like single or double integrators (see [5, 1]). An excellent review can be found in [4]. The consensus results for the case where the agents have a general, yet identical, linear dynamics with time-independent communication topology are reported in [2].

Despite the extensive amount of research available in the context of consensus/synchronization of multi-agent systems, there are few works which have considered network of agents with general linear dynamics together with time-dependent communication topology. This is considered in [6], and a consensus protocol is proposed for possibly time-varying communications. However, there in, the agents are not allowed to have exponentially unstable dynamics. Assuming nonzero dwell time for switching among possible communication graphs, consensus problem for multi-agent systems with general linear dynamics is studied in [8].

In this note, we consider the network of agents with general, but identical, linear dynamics, and the communication topology may switch within a finite set of admissible topologies. We allow switching to be arbitrary meaning that the switching may occur at arbitrary time instances. It will be observed that synchronization is essentially an output stability problem. Consequently, given the agents' dynamics, coupling rule, and set of admissible topologies, we derive conditions under which synchronization is achieved.

Preliminaries. Let $G_i = (V, E_i)$ with $i = 1, 2, \ldots, N$ be an undirected (unweighted) graphs where $V = \{1, 2, \ldots, p\}$ is the vertex set and $E_i \subseteq V \times V$ are the edge sets. A diffusively coupled multi-agent system with switching topology consists of a collection of identical linear input/state/output systems given by

\begin{align}
\dot{x}_j(t) &= Ax_j(t) + Bu_j(t) \\
y_j(t) &= Cx_j(t)
\end{align}

(1a) (1b)

together with the diffusive coupling rule

\begin{align}
u_j(t) &= \sum_{(i,j) \in E_x(i)} (y_i(t) - y_j(t)),
\end{align}

(1c)
where \( j \in V, x_j \in \mathbb{R}^n \) is the state of agent \( j \), \( \sigma \) is a right-continuous piecewise constant function of time taking its value from the index set \( \{1, 2, \ldots, N\} \), and \( u_j \in \mathbb{R}^m \) is the diffusive coupling term. For each \( i = 1, 2, \ldots, N \), let \( L_i \) denote the Laplacian matrix corresponding to the graph \( G_i = (V, E_i) \). Then, the multi-agent system (1) can be written in compact form as

\[
\dot{x}(t) = A_{\sigma(t)} x(t),
\]

where \( x = \text{col}(x_1, \ldots, x_p) \), and \( A_{\sigma(t)} := I_p \otimes A - L_{\sigma(t)} \otimes BC \), where \( \otimes \) denotes the Kronecker product.

We say the multi-agent system (1) is synchronized if every solution of (2) satisfies

\[
\lim_{t \to \infty} (x_j(t) - x_k(t)) = 0 \text{ for all } j, k = 1, 2, \ldots, p.
\]

**Main Contribution.** As mentioned earlier, in this note, conditions to guarantee the synchronization of network (2) are provided. The proposed conditions are mostly derived from the following crucial result.

**Theorem 2.** Consider the network (2). Let the eigenvalues of \( L_i \) be denoted as \( 0 = \lambda_i^1 \leq \lambda_i^2 \leq \lambda_i^3 \leq \ldots \leq \lambda_i^p \) for each \( i = 1, 2, \ldots, N \). Let \( \underline{\lambda} \) and \( \overline{\lambda} \) denote the minimum and maximum values of \( \lambda_i^j \) for \( i \in \{1, 2, \ldots, N\} \) and \( j \in \{2, 3, \ldots, p\} \), respectively. Then, the network (2) is synchronized if there exists a common quadratic Lyapunov function (CQLF) for the pair of matrices \( A - \underline{\lambda} BC \) and \( A - \overline{\lambda} BC \).

**Remark.** For the special case where the diffusive coupling term \( u_j \) in (1c) is a scalar function, the multi-agent system (1) is synchronized if the matrices \( A - \underline{\lambda} BC \) and \( A - \overline{\lambda} BC \) are both Hurwitz, and their product does not have any negative real eigenvalues. This can be verified based on Theorem 2 and a result available on the existence of CQLF in [7].

According to Theorem 2, the network (2) is synchronized if there exists a CQLF for the pair of matrices \( A - \underline{\lambda} BC \) and \( A - \overline{\lambda} BC \). There are sufficient conditions for the existence of a CQLF based on commutativity, simultaneous triangularizability, and solvable Lie algebra (see [3]). However, these conditions imply direct constraints on the matrices \( A \) and \( BC \), namely \( A \) and \( BC \) must commute or at least be simultaneously triangularizable. We avoid imposing such constraints and, instead, propose sufficient conditions which depend not only on dynamics of the agents but also the topology of the network.

**Theorem 2.** The network (2) is synchronized if there exists indices \( k \in \{1, 2, \ldots, N\} \) and \( \ell \in \{2, 3, \ldots, p\} \) such that \( A - \lambda_k^\ell BC \) is Hurwitz and

\[
\max(\lambda_k^\ell - \underline{\lambda}, \overline{\lambda} - \lambda_k^\ell) \|H_k^\ell\|_\infty < 1
\]

where

\[
H_k^\ell = C(sI - A + \lambda_k^\ell BC)^{-1}B.
\]

**REFERENCES**


Synchronization analysis of multi-agent systems


