A GEOMETRIC SUBGRADIENT DESCENT ALGORITHM FOR THE ECONOMIC LOAD DISPATCH PROBLEM *

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1. Introduction. The economic load dispatch problem (ELDP) is a classical problem in the power systems community. It consists in the optimal (minimal cost) scheduling of the outputs p_i of n power generating units to meet the required load demand p_d, subject to bound inequalities and balance equality constraints:

$$\min_{\mathbf{p}\in\mathbb{R}^n} f_T(\mathbf{p}) = \sum_{i=1}^{n} a_i p_i^2 + b_i p_i + c_i + \left| d_i \sin\left[e_i \left(p_i^{\min} - p_i \right) \right] \right|, \quad \mathbf{a},\mathbf{b},\mathbf{c},\mathbf{d},\mathbf{e} \in \mathbb{R}_+^n,$$

$$\text{s.t.} \begin{cases} p_i^{\min} \leq p_i \leq p_i^{\max}, & \text{(bounds on each unit)} \\ \sum_{i=1}^n p_i = p_d + p_l, \quad p_d \in \mathbb{R}_+ & \text{(balance: respect demand + loss)} \\ p_l = \sum_{i=1}^n \sum_{j=1}^n p_i B_{ij} p_j + \sum_{i=1}^n b_i^0 p_i + b^{00}, \end{cases}$$

where $\mathbf{B} \in \mathbb{R}^{n\times n}$, $\mathbf{b}^0 \in \mathbb{R}^n$ and $b^{00} \in \mathbb{R}$ describe the power loss characteristics. The total cost function is thus the sum of the individual units' costs. The sine term is included to take into account the discontinuities in the incremental heat curves caused by the admission valves of the steam turbines. This phenomenon is referred to as the valve-point effect in the literature. This optimization problem is challenging on three different levels: the geometry of its feasible set, the non-differentiability of its cost function and the multimodal aspect of its landscape. For this reason, the ELDP has received much attention in the past few years and several derivative-free optimization techniques have been proposed to tackle its multimodal and non-differentiable characteristics (see *e.g.* [1] and the references therein). However, all these heuristic algorithms face difficulties to respect the load equality constraint, trying to impose it using penalty terms, Lagrangian multipliers or direct elimination. Furthermore, these techniques are not endowed with convergence guarantees.
In this work, we propose a different approach exploiting the rich geometrical structure of the problem. We show that the equality constraint can be handled in the framework of Riemannian manifolds, and we develop a projected subgradient descent algorithm to provide fast and robust convergence to local minima. Details can be found in [2].

2. Projected subgradient descent for the ELDP. As depicted in Figure 1, the equality constraint defines the surface of an ellipsoid in $\mathbb{R}^n$, which is a smooth embedded manifold. The canvas of optimization on manifolds allows to produce a sequence of iterates that satisfy the equality constraint at all times [3]. Consequently, the original problem is turned into a simpler bound-constrained problem. In this

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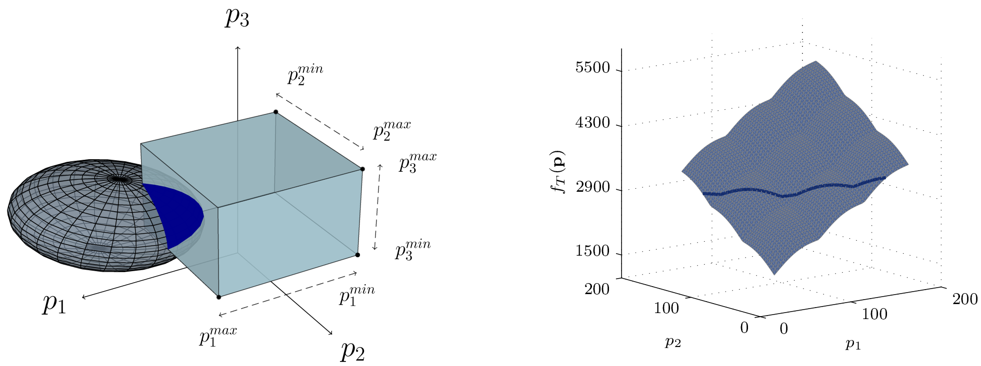

FIG. 1.1. *(left) Illustration of the feasible set (dark blue) for $n = 3$. (right) Example of optimization landscape for $n = 2$.*

respect, we develop the necessary tools specifically for the ellipsoid manifold.
Next, we study the cost function and show that it is piecewise smooth. This suggests to use Clarke's generalized calculus to obtain the associated subgradient. Although this set can be hard to compute in general, we show that the cost function at hand can be expressed as the pointwise maximum of smooth functions; as a result, the subgradient is easily available, following the framework presented by Dirr *et al.* in [4]. We then describe how a deterministic descent direction can be obtained by solving a simple, low-dimensional quadratic program. Finally, we use the result of this subproblem to perform a classical descent iteration using line search with Armijo's rule.

3. Experimental validation. We tested our approach on 4 real data sets of dimensions 3, 5, 6 and 15. We also implemented some state-of-the-art heuristics with good global search performance, namely, cross-entropy, particle swarm optimization and differential evolution. The experimental results indicate that our local method surpasses the heuristics both in terms of local convergence speed and quality. Furthermore, our approach presents the major advantages of scaling well with respect to the problem dimension and producing valid iterates at all times. On the other hand, the heuristics provide better exploration of the search space. Therefore, we propose and test an algorithm that offers the best of both worlds by combining our method with the differential evolution method. In future work, we will consider combining our method with Particle Swarm Optimization, a global optimization tool that was previously adapted to a manifold setting [5].

REFERENCES

[1] J. Vlachogiannis and K. Lee, *Economic Load Dispatch - A Comparative Study on Heuristic Optimization Techniques With an Improved Coordinated Aggregation-Based PSO*, IEEE Transactions on Power Systems, vol. 24, no. 2, pp. 991–1001, 2009.

[2] P. B. Borckmans, S. Easter Selvan, N. Boumal and P.-A. Absil, *A Riemannian Subgradient Algorithm for Economic Dispatch with Valve-Point Effect*, submitted, 2012.

[3] P.-A. Absil, R. Mahony, and R. Sepulchre, *Optimization Algorithms on Matrix Manifolds*, Princeton University Press, 2008.

[4] G. Dirr, U. Helmke, and C. Lageman, *Nonsmooth Riemannian Optimization with Applications to Sphere Packing and Grasping*, in Lecture Notes in Control and Information Sciences, vol. 366, pp. 29–45, Springer, 2007.

[5] P. B. Borckmans and P.-A. Absil, *Oriented Bounding Box Computation Using Particle Swarm Optimization*, in Proceedings of ESANN 2010, Bruges, pp. 345-350, 2010.