OPTIMAL DESIGN OF AN UNDERGROUND MINE DECLINE WITH AN ASSOCIATED VENT RAISE*

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Abstract. In many underground mines, access for equipment and personnel is provided by means of a network of declines, and air is extracted from the network via vertical vent raises using surface fans. Current industry practice is to design the layout of the decline network first and then to add the vent raises. This paper presents a method for optimally designing a decline together with its associated vent raise. We establish a sufficient condition on the optimal (least cost) location of the vent raise with respect to certain fixed points along the decline, for the case where there is an upper bound on the gradient of the decline. This result yields a procedure for generating a near-optimal design for a gradient-constrained decline and an associated vent raise in the presence of additional constraints such as an upper bound on the curvature of the decline.

Key words. underground mine design, mine ventilation, minimum bounding circle, minimax

AMS subject classifications. 49K35, 86A60, 90B10

1. Introduction. In many underground mines, access between the surface and the working areas of the mine is provided by means of a network of declines and drives. This network provides access for equipment and personnel and is used by the trucks that haul the ore to the surface.

The problem of designing decline networks in an optimal manner has received some attention in recent years [6, 3, 4, 5]. The optimality criterion used in these studies is to minimize the total life-of-mine cost of the network, which comprises the cost of constructing the declines and the cost of hauling the ore. The optimization is subject to a number of constraints. The ore trucks must be able to navigate the declines; navigability constraints include upper bounds on the gradient and curvature of the declines and may include other constraints. Additionally, the decline network must avoid certain regions such as the ore body and geologically unstable regions. Research conducted in this area has resulted in the development of the software application DOT (Decline Optimization Tool). DOT can be used to rapidly generate alternative strategic designs for a decline network, from which a mining engineer can select a design to use as a template for a final detailed design.

An essential part of the task of planning an underground mine is to design the ventilation network that provides fresh air for personnel and equipment and removes dust and other contaminants. The main (primary) ventilation is typically provided by surface fans that extract the air from the decline network via vertical vent raises. Current industry practice is to design the access network first (using formal optimization methods or otherwise) and then to add the vent raises to the design.

The present paper represents a first attempt at generating an optimal design for the decline network and the ventilation infrastructure concurrently. The potential for generating better designs in this way should be evident. There is often a substantial amount of freedom available in specifying the path in space of a decline joining two points at a specified gradient, and all such paths have the same length. If the decline

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network is designed without regard for the need to add the vent raises then it is likely that longer connectors between the declines and the vent raises will be required. By varying the declines and vent raises in ways that allow them to approach each other more closely where they need to be connected, it should be possible to reduce the total length of the network and hence its construction and operational cost. The cost of the ventilation network is significant; in one case study undertaken, the cost of constructing the ventilation represented between 25 and 30 percent of the total cost of the ventilation and access network including all construction and haulage costs (but excluding ventilation operational costs).

The aim of the present paper is to develop an algorithm for generating an approximately optimal (least cost) design for a single decline with an associated vent raise where there is an upper bound on the gradient of the decline. In Section 2 we state the problem formally and present an initial attempt at a solution. In Section 3 we propose an alternative approach to the problem which is more promising from the point of view of generating a practical algorithm. In Section 4 we develop the mathematical theory and outline an algorithm for generating a near-optimal design. The concluding section contains a summary of the findings and proposals for further research.

2. The problem and an initial solution. We consider a simple geometric design for the decline access and primary ventilation in an underground mine. Specifically, we investigate the problem of optimally designing a decline and a single vent raise that is connected to it at a number of points that are uniformly spaced vertically as shown in Figure 2.1. In practice, each connection point would be controlled by a regulator (door), and only some of the regulators would be open at any given time. Air enters the mine via the decline and is extracted through the vent raise by surface fans. Access to the ore is provided by drives that break out from the decline at specified access points which we assume are uniformly spaced vertically, with the same spacing as the vent raise connection points. The assumption of uniformly spaced access points is typically satisfied, at least approximately, in mines where the mining method used is sublevel open stoping or a related method. The spacing of the vent raise connection points uniformly is reasonable in order to efficiently ventilate all parts of the mine. It follows that there is one connection point between each pair of adjacent access points. The drives are ventilated by means of a secondary (ducted) ventilation system which does not enter into the present problem. It is desirable that the vent raise be vertical or nearly so, in order to make it as short as possible so as to reduce construction costs and energy losses due to resistance and to minimize the shock losses that occur at bends.

We will make some simplifying assumptions. In practice, the vent raise would be offset from the decline by a short distance and connected to it by a horizontal passage at each connection point. We ignore this detail here and assume that the vent raise passes through the decline at the connection points. We assume that the decline has a given constant gradient; this can generally be achieved in practice provided that the access points are not displaced too far horizontally with resect to one another. We do not impose any constraint on the curvature of the decline. With these assumptions, the length of the decline between any two points with given heights is determined. Thus, the costs of constructing the decline and hauling the ore along it, both of which are proportional to the length of the decline, are fixed and need not enter into the optimization. The problem is to design the decline and the vent raise so that the vent raise is as near to vertical as possible.

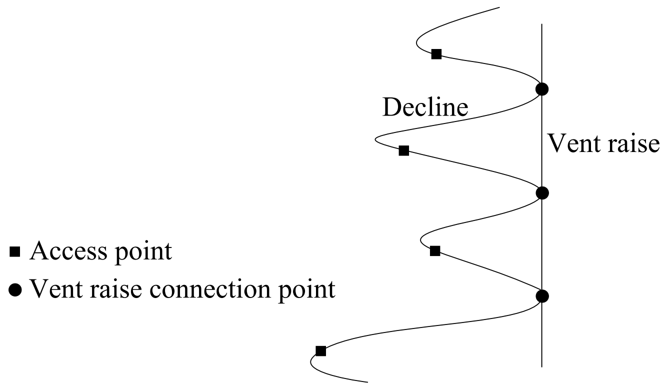

FIG. 2.1. *A decline with an associated vent raise.*

The material in the remainder of this section draws partly on the account given in the unpublished report by Kutadinata [2].

We consider a decline with n access points, $A_1, \ldots, A_n$, and $n-1$ vent raise connection points, $V_1, \ldots, V_{n-1}$. The $2n-1$ points are arranged along the decline in the order $A_1, V_1, \ldots, A_{n-1}, V_{n-1}, A_n$, from the bottom upward. The coordinates of the access points are given and the coordinates of the vent raise connection points are to be determined. The vertical distances from A_i to A_{i+1} and from V_i to V_{i+1} are equal to a constant, h, for all valid values of i.

Let m denote the gradient of the decline. Then the length, k, of the horizontal projection of each link of the decline between two adjacent access points is given by $k = h/m$. Assume that a vertical vent raise can be constructed subject to these constraints. Let V denote the common location of the horizontal projection of all of the vent raise connection points. With a slight abuse of notation, we now use $A_1, \ldots, A_n$ to denote the horizontal projections of the access points. Note that the projections of the access points need not all be distinct, because access points may be vertically aligned. Then the horizontal projection of the decline is a concatenation of paths of length k from A_i to A_{i+1} via V, $1 \leq i \leq n-1$, such that the length of each loop from V to A_i and back to V is also k. Equivalently, there is a number s such that the length of the path from A_i to V is s and the length of the path from V to A_{i+1} is $k-s$, for all i such that $1 \leq i \leq n-1$.

In what follows, $\|XY\|$ denotes the distance from X to Y in the 2-dimensional Euclidean metric.

The following theorem establishes the region of the plane in which V must lie. We present it here without proof.

THEOREM 2.1. *Let $A_1, \ldots, A_n$ $(n \geq 2)$ and V be points in the plane. Let k be a number such that $k > \|A_iA_j\|$ for all i and j, $1 \leq i \leq j \leq n$. For each value of i and j such that $1 \leq i \leq j \leq n$, let E_{ij} denote the closed region bounded by the ellipse with foci at A_i and A_j and with major axis equal to k. Let E be the intersection of all of the elliptical regions of the forms E_{1i} $(2 \leq i \leq n)$, E_{in} $(2 \leq i \leq n-1)$, and E_{ii} $(2 \leq i \leq n-1)$. Then the following statements are equivalent:*

1. *There is a number s, $0 \leq s < k$, such that for each i, $1 \leq i \leq n-1$, there exists a path from A_i to V in the plane with length s, and for each j, $2 \leq j \leq n$, there exists a path from V to A_j in the plane with length $k-s$;*
2. *$V \in E$.*

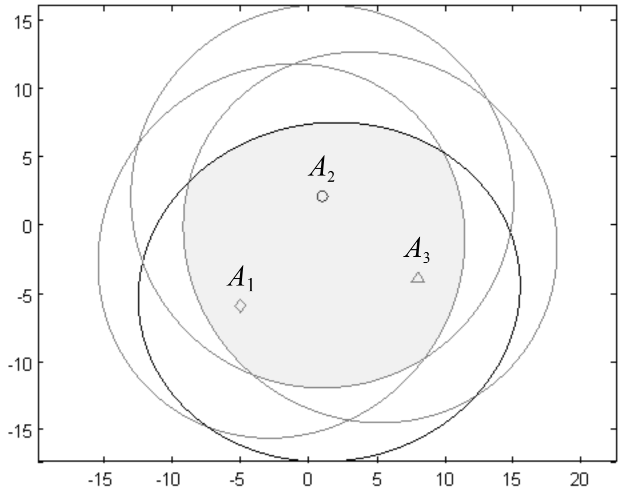

FIG. 2.2. *An example of the solution space with three access points.*

The region E is the intersection of the closed regions bounded by $n-2$ circles and $2n-3$ (other) ellipses. The simplest non-trivial case is $n = 3$, for which there are three ellipses and one circle. This case is illustrated in the MATLAB plot in Figure 2.2 which is reproduced from [2]. In general, a vertical vent raise can be constructed provided that E is non-empty.

When once a location for V has been selected in E, a value for s in Theorem 2.1 can be calculated using $s = \max_{1\le i\le n-1} \|A_iV\|$. The vertical coordinate of each vent raise connection point can then be found by adding sm to the vertical coordinate of the access point immediately below it.

3. An alternative approach to the problem. The approach described above has some drawbacks from the point of view of developing an algorithm. It determines a feasible region rather than a specific point, and it gives no clue as to how best to select the point in the region where the vent raise should be located, especially if other constraints must also be met. Further, it does not specify a location for the vent raise if the region is empty.

An alternative approach which addresses these issues is as follows. As before, let $A_1, \ldots, A_n$ be the horizontal projections of the access points. The gradient of the decline is still assumed to be constant but only an upper bound for its value, m_0, is given. Let V be the horizontal projection of the vertical vent raise. Then there is a constant, k, such that for all pairs of values of i and j for which $1 \le i \le n-1$ and $2 \le j \le n$, the sum of the lengths of the two directed subpaths from A_i to V and from V to A_j in the projected decline equals k. Therefore, $k \ge \|A_iV\| + \|A_jV\|$ for all such i and j and so $k \ge T(V)$ where $T(V) = \max_{i,j}\{\|A_iV\| + \|A_jV\|\}$. However, k is also constrained by the requirement that $k \ge h/m_0$ and so $k \ge \max\{h/m_0, T(V)\}$. Conversely, given any value of k that satisfies this inequality, a decline can be found using that value of k that satisfies all of the constraints.

In order to minimize the length of the decline, and hence its construction cost, we want to design a decline using the value $k = \max\{h/m_0, T(V)\}$. Further, V should be chosen to minimize $T(V)$ to ensure that k is as small as possible.

Let V_0 be a value of V that minimizes $T(V)$. Then three cases can arise:

1. $T(V_0) < h/m_0$. Then $k = h/m_0$ and the upper bound on the gradient is achieved. Some other values of V (not necessarily values that minimize $T(V)$) yield the same value for k, and so there is some flexibility available in selecting the location of the vent raise without increasing the length of the decline. This added flexibility could be used to meet additional constraints on the decline such as a curvature constraint.

2. $T(V_0) = h/m_0$. Then $k = h/m_0 = T(V_0)$ and the upper bound on the gradient is achieved. The vent raise is limited to the location or locations for which $T(V)$ is minimized.

3. $T(V_0) > h/m_0$. Then $k = T(V_0)$ and the upper bound on the gradient is not achieved. As in the previous case, the vent raise is limited to the location or locations for which $T(V)$ is minimized. If the gradient falls substantially short of the upper bound then the best practical solution to the problem might be obtained by relaxing the requirement that the vent raise be strictly vertical so as to reduce the length of the decline in the final design; however, a design with a vertical vent raise will still provide a useful starting point.

The first two cases correspond to the situation where the solution space obtained in Section 2 is non-empty.

The vertical coordinate of the vent raise connection point between A_i and A_{i+1} $(1 \le i \le n-1)$ may be determined by adding $h\|A_iV_0\|/\left(\|A_iV_0\| + \|A_{i+1}V_0\|\right)$ to the vertical coordinate of A_i.

4. Mathematical derivations. In this section we state and prove a sufficient condition for a point V_0 to minimize T and we explain how this result can be used in a practical algorithm for optimally designing a decline with an associated vent raise.

Given a list of n points in the plane (where $n > 1$ to avoid certain trivialities), the *minimum bounding circle* (MBC) is the smallest circle that contains all of the points either on or inside it.

The following properties of the MBC can be readily shown:

1. The MBC for any given list of points is unique.

2. At least two of the given points are on the MBC, and the angles of the arcs between any pair of adjacent such points cannot exceed π.

3. The centre of the MBC is the point that minimizes the maximum distance to each of the given points.

THEOREM 4.1. *Let $A_1, \ldots, A_n$ be n points in the plane $(n > 1)$. Let X be the centre of the MBC of $A_1, \ldots, A_n$. Let $T(V) = \max_{i,j}\{\|A_iV\| + \|A_jV\|\}$ where $1 \le i \le n-1$ and $2 \le j \le n$. Then X minimizes T.*

Proof. Let r be the radius of the MBC. Then $\|A_iX\| \le r$ for all i, so $\|A_iX\| + \|A_jX\| \le 2r$ for all i and j and hence $T(X) \le 2r$. Let Y be any point that is different from X. In order to establish the theorem, it is sufficient to show that $T(Y) \ge 2r$. Since the MBC is unique, Y is not the centre of an MBC and so there exists i such that $\|A_iY\| > r$. Three cases arise.

Case 1: $\|A_iY\| > r$ for some i such that $2 \le i \le n-1$. Then $T(Y) \ge 2\|A_iY\| > 2r$.

Case 2: $\|A_1Y\| > r$ and $\|A_nY\| > r$. Then $T(Y) \ge \|A_1Y\| + \|A_nY\| > 2r$.

Case 3: $\|A_1Y\| > r$ and $\|A_iY\| \le r$ for $2 \le i \le n$, or $\|A_nY\| > r$ and $\|A_iY\| \le r$ for $1 \le i \le n-1$. In the argument that follows we assume the former; the argument for the latter case is similar. Let C be the circle of radius r centred at Y as shown in Figure 4.1. Then $A_2, \ldots, A_n$ are on or inside C. Since the angle of the arc of the MBC that falls outside C is greater than π, A_1 is on that arc. Let B be the point

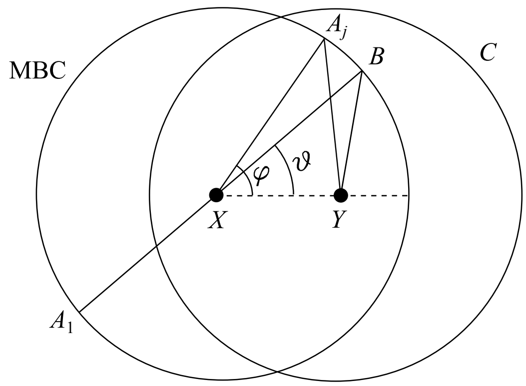

FIG. 4.1. *Case 3 of Theorem 4.1.*

diametrically opposite A_1 on the MBC. Then B is on or inside C, since otherwise the arc of the MBC from A_1 through B to C would turn through more than π and would contain no points from $A_2, \ldots, A_n$, violating Property 2 of the MBC. Again by Property 2, the two half-open semicircles of the MBC between A_1 and B that include the point B must each contain at least one of the points $A_2, \ldots, A_n$. In particular, there is a point A_j $(2 \leq j \leq n)$ on one of the semicircles such that either A_j and Y are on opposite sides of the line through A_1 and B or at least one of A_j and Y is on that line. Let $\vartheta = \angle YXB$ and $\varphi = \angle YXA_j$. From triangles (or degenerate triangles) XYB and XYA_j we obtain $\|YB\|^2 = \|XY\|^2 + r^2 - 2\|XY\|r\cos\vartheta$ and $\|YA_j\|^2 = \|XY\|^2 + r^2 - 2\|XY\|r\cos\varphi$ respectively. Since $\vartheta \leq \varphi$, it follows that $\|YB\| \leq \|YA_j\|$. Thus $T(Y) \geq \|A_1Y\| + \|A_jY\| \geq \|A_1Y\| + \|YB\| \geq \|A_1B\| = 2r$.

We have established that $T(Y) \geq 2r$ in all cases and so X minimizes T. □

It can be shown that points other than the centre of the MBC can also minimize T in some situations. The details are not given here.

A number of algorithms have been developed for finding the MBC of a given set of points. One such algorithm is Welzl's algorithm [1], which is straightforward to implement and runs in time $O(n)$ on average, where n is the number of points.

The following procedure can be used for designing a decline and an associated vent raise, given the locations of the access points:

1. Find the centre of the MBC of the horizontal projections of the access points.
2. Calculate the vertical coordinates of the connection points.
3. Use a numerical optimization method such as simulated annealing to modify the horizontal coordinates of each connection point independently to accommodate the curvature constraint and any other constraints.
4. Design the decline through the access points and the connection points using a currently available algorithm such as the algorithm used in DOT.
5. Design the vent raise by joining the connection points with straight lines.

The resulting vent raise will in general not be exactly vertical but it can be expected to be nearly so.

5. Conclusion. We have established a result that yields a practical algorithm for finding a near-optimal design for a gradient-constrained decline with an associated vent raise. Future work might focus on making the design more realistic by adding short horizontal drives from the decline to the vent raise at the connection points. Since the lengths of the drives are permitted to vary within certain limits, this approach would provide added flexibility that would allow the vent raise to be made even closer to vertical.

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