PASSIVITY BASED CONTROL OF UNDERACTUATED 2-D SPIDERCRANE MANIPULATOR

G.GOGTE*, VENKATESH C.†, F.KAZI‡, N.M.SINGH§, AND R.PASUMARTHY¶

Abstract. This paper proposes a feedback passivation method for the bio-inspired 2-D SpiderCrane system. The property that the equilibrium point of a passive system is always stable is used to ensure the stability of the payload. We employ decoupling of cable and pulley dynamics in the SpiderCrane mechanism. Further, the payload is viewed as a pendulum whose suspension point lies on a mass that moves in a 2 dimensional space. We approximate the cable pylons structure as integrator systems which can be interconnected to the pendulum (which is a passive system) so that the resultant feedback connection gives a passive system. The forces which actuate the system are evaluated from nonlinear feedbacks, using a suitable energy function. This method is suitable for underactuated systems and does not require the systems to be in port-controlled Hamiltonian form. The idea of this paper is to obtain the required system dynamics by designing a suitable feedback rather than energy shaping of the system.

Key words. Passivity, Underactuated Manipulators, Bio-inspired Robotics

AMS subject classifications. 93C10, 93D15

1. Introduction.

Cranes form an important class of nonlinear systems. The control and stability problem of overhead cranes differs from other nonlinear mechanical systems like robot manipulators due to negative damping effect produced by length variations during operations. The system considered in this paper is the two dimensional SpiderCrane system developed by Laboratory of Automatic Control at École Polytechnique Féderale de Lausanne (EPFL). This design was imagined in order to reduce the time involved in carrying loads for the classical cranes by minimizing the inertia associated with classical cranes and replicates spider like motion.

Most crane operators move the load with the cable almost vertical. Very few operators venture to shift the upper trolley in anticipation of the swing and the desired final payload position. The problem is to achieve fast and precise payload positioning while minimizing the swing. This problem has been solved by various approaches like the controlled Lagrangian approach [1], flatness based approach [2], receding horizon control [3], interconnection and damping assignment passivity based control (IDA-PBC) [4], [5]. The IDA-PBC methodology is also a passivity based design applicable to systems that can be modelled in the port-controlled Hamiltonian form [6], [7]. This method carries out energy shaping of the the system to transform a given port-controlled Hamiltonian system into another port-controlled Hamiltonian system with some desired properties.

This paper proposes a passivity based controller to achieve precise load positioning with minimum swing. The payload is viewed as a pendulum which is a passive system. We interconnect it suitably with the pylon and pulley systems which are also passive to obtain another passive system.

2. SpiderCrane system.

A 2-D SpiderCrane system consists of two fixed verical support structures called pylons. A pulley is mounted on top of each pylon. The

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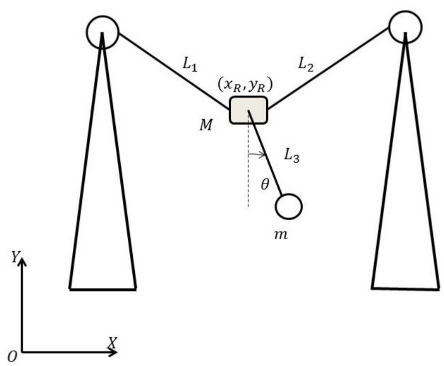
lengths of the cables L_1 and L_2 suspended from the pylons are variable. A ring of mass M is attached at the end of the cables, from which the payload is suspended. Positions of the payload can be changed by changing the cable lengths L_1 and L_2.

The payload suspended from the ring can be approximated to a pendulum of mass m and fixed length L_3. It's angular position with respect to the vertical (as shown in Fig 1) is given by the angle θ such that $\theta \in [0, 2\pi)$ and its position in the X-Y plane is given by (x_R, y_R) where $x_R \in \mathbb{R}^1$ and $y_R \in \mathbb{R}^1$. A detailed mathematical model of this system can be found in [8].



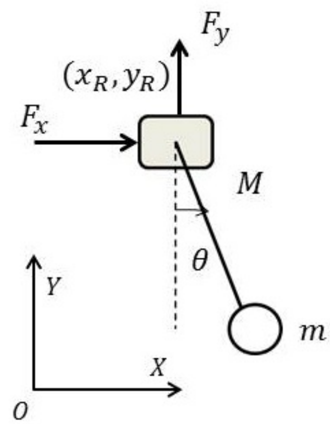


Fig 1:Schematic of SpiderCrane

The Euler-Lagrange equations for the system are:

$$F_x = (M+m)\ddot{x}_R + mL_3\cos\theta\ddot{\theta} - mL_3\sin\theta\dot{\theta}^2 \tag{2.1}$$

$$F_y = (M+m)\ddot{y}_R + mL_3\sin\theta\ddot{\theta} + mL_3\cos\theta\dot{\theta}^2 + (M+m)g \tag{2.2}$$

$$0 = mL_3\cos\theta\ddot{x}_R + mL_3\sin\theta\ddot{y}_R + mL_3^2\ddot{\theta} + mgL_3\sin\theta \tag{2.3}$$

Here F_x and F_y are the forces in the horizontal (x) and the vertical (y) directions respectively. The control $u \in \mathbb{R}^2$ is defined as $u = [u_x \; u_y]^T$. The control objective is to move the payload from some initial position (x_i, y_i) to some desired position (x_D, y_D). At rest the system satisfies the condition $\theta = 0$. The input u is applied to manipulate the (x_R, y_R) coordinates of the ring and not the payload. Thus the payload position θ is unactuated. The system is underactuated since there are three degrees of freedom x_R, y_R and θ while two inputs u_x and u_y.

3. Passivity Theory. Passivity is a useful tool for the analysis of nonlinear systems [9]. It relates well to the Lyapunov stability method. A system $y = h(t, u)$ is passive if $u^T y \geq 0$. Consider a dynamical system given by: $\dot{x} = f(x, u)$ and $y = h(x, u)$ where $f : \mathbb{R}^n \times \mathbb{R}^p \rightarrow \mathbb{R}^n$ is locally Lipschitz, $h : \mathbb{R}^n \times \mathbb{R}^p \rightarrow \mathbb{R}^n$ is continuous, $f(0, 0) = 0$ and $h(0, 0) = 0$. The system is said to be passive if there exists a positive semidefinite function $V(x)$ such that

$$u^T y \geq \dot{V} = \frac{\partial V}{\partial x} f(x,u) \tag{3.1}$$

A passive system has a stable origin. Any system may be stabilized by using a suitable feedback that forces the system to be passive. Consider the following system: $\dot{x} = f(x,u)$, $y = h(x)$. Assume that the system is:
1. Passive with a radially unbounded positive definite storage function.
2. Zero state detectable.
Then $x = 0$ can be globally stabilized with $u = -\varphi(y)$ where φ is any locally Lipschitz function such that $\varphi(0) = 0$ and $y^T \varphi(y) > 0, \forall y \neq 0$.

The feedback and parallel interconnection of two or more passive systems is always passive. The proof of these properties can be found in [10]. Consider the system represented by: $\dot{x} = f(x) + G(x)u$. Suppose there exists a radially unbounded positive definite C^1 function $V(x)$ such that $\frac{\partial V}{\partial x} f(x) \leq 0 \ \forall x$. Assume an output: $h(x) = [\frac{\partial V}{\partial x} G(x)]^T$. Therefore $\dot{V} = \frac{\partial V}{\partial x} f + [\frac{\partial V}{\partial x} G(x)]^T] u \leq y^T u$. Thus we get a system with input u and output y which is passive and hence stabilizable. A feedback can be introduced in the system so that the overall system is passive. These features of passive systems will be used for controller design as shown in the next section.

4. Controller Design. The payload approximated to a pendulum needs to be stabilized. We need to design a feedback that makes the interconnection of the payload, cables and the pylons passive. The interconnection may be imagined as in Fig 2.

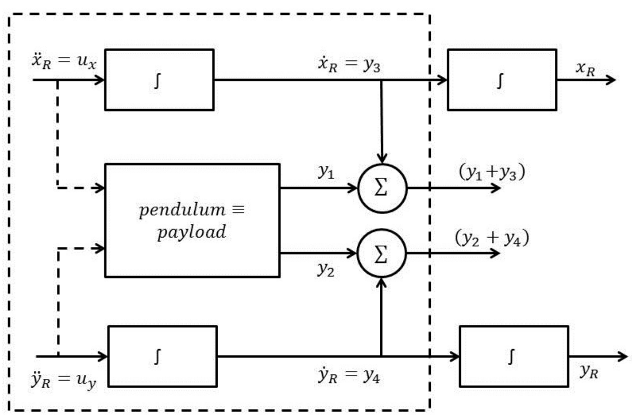


Fig 2: Spider crane as interconnection of passive systems

The cables and pylons are approximated to integrator systems. There are two integrator systems; input for one is the acceleration in the horizontal direction i.e. $\ddot{x}_R$ and its output is velocity in the horizontal direction i.e. $\dot{x}_R$. Similarly the input for the second integrator is the acceleration in the vertical direction $\ddot{y}_R$ and its output is the velocity in the vertical direction $\dot{y}_R$. Hence we get two single integrator systems corresponding to velocities in the horizontal and vertical directions respectively which are perfectly passive. The storage functions for the systems are $\frac{1}{2}\dot{x_R}^2$ and $\frac{1}{2}\dot{y_R}^2$ respectively.

The system visualization as as done in Fig(2) is on the basis of the concept of collocated partial linearization [11], [12]. This property holds for the entire class of underactuated systems.

Consider the actuated variables x_R and y_R as outputs. These outputs are collocated with inputs u_x and u_y respectively. We consider the total forces in the system F_x and F_y as the sum of the force exerted by the actuators i.e. $\tilde{F_x}$ and $\tilde{F_y}$ and the forces resulting from pendulum dynamics and gravity i.e. $\alpha_1(\theta, \dot{\theta})$ and $\alpha_2(\theta, \dot{\theta})$ in the horizontal and vertical directions respectively.

$$F_x = \tilde{F_x} + \alpha_1(\theta, \dot{\theta}) \tag{4.1}$$
$$F_y = \tilde{F_y} + \alpha_2(\theta, \dot{\theta}) \tag{4.2}$$

Comparing (4.1) with (2.1) and (4.2) with (2.2)

$$\tilde{F_x} = (M + m)\ddot{x}_R \tag{4.3}$$
$$\tilde{F_y} = (M + m)\ddot{y}_R \tag{4.4}$$

$$\alpha_1(\theta, \dot{\theta}) = mL_3 \cos\theta\ddot{\theta} - mL_3 \sin\theta\dot{\theta}^2 \tag{4.5}$$
$$\alpha_2(\theta, \dot{\theta}) = mL_3 \sin\theta\ddot{\theta} + mL_3 \cos\theta\dot{\theta}^2 + (M + m)g \tag{4.6}$$

Therefore

$$\frac{\tilde{F_x}}{(M + m)} = \ddot{x}_R \tag{4.7}$$
$$\frac{\tilde{F_y}}{(M + m)} = \ddot{y}_R \tag{4.8}$$

Let $\frac{\tilde{F_x}}{(M+m)} = u_x$ and $\frac{\tilde{F_y}}{(M+m)} = u_y$ i.e.

$$u_x = \ddot{x}_R \tag{4.9}$$
$$u_y = \ddot{y}_R \tag{4.10}$$

From (2.3) we can write:

$$\ddot{\theta} = -\frac{1}{L_3}(\cos\theta\ddot{x}_R + \sin\theta\ddot{y}_R + g\sin\theta) \tag{4.11}$$

Substituting $\ddot{x}_R = u_x$ and $\ddot{y}_R = u_y$ in (4.11)

$$\ddot{\theta} = \frac{-1}{L_3}(\cos\theta u_x + \sin\theta u_y + g\sin\theta) \tag{4.12}$$

Now energy of the pendulum E_p is given as:

$$E_p = \frac{1}{2}mL_3^2\dot{\theta}^2 - mgL_3\cos\theta \tag{4.13}$$

$$\dot{E_p} = -mL_3\dot{\theta}\cos\theta u_x - mL_3\dot{\theta}\sin\theta u_y \tag{4.14}$$

Let $y_1 = -mL_3\dot{\theta}\cos\theta$ and $y_2 = -mL_3\dot{\theta}\sin\theta$.
The total system's storage function is given by the sum of the energy of pendulum

and the storage functions of each of the integrator systems i.e systems having input as acceleration and output as velocity.

$$\widetilde{E} = E_p + \frac{1}{2}\dot{x}_R^2 + \frac{1}{2}\dot{y}_R^2 \tag{4.15}$$

$$\dot{\widetilde{E}} = y_1 u_x + y_2 u_y + \dot{x}_R \ddot{x}_R + \dot{y}_R \ddot{y}_R \tag{4.16}$$

Here $\dot{x}_R = y_3$ and $\dot{y}_R = y_4$.

$$\dot{\widetilde{E}} = (y_1 + y_3)u_x + (y_2 + y_4)u_y \tag{4.17}$$

We select an energy function V as:

$$\begin{aligned} V = \widetilde{E} + \frac{1}{2}K_{p_1}[\int y_1 dt + (x_R - x_D)]^2 \\ + \frac{1}{2}K_{p_2}[\int y_2 dt + (y_R - y_D - L_3)]^2 \end{aligned} \tag{4.18}$$

The term L_3 i.e. pendulum length has been used in (4.18) to specify the position of payload mass in terms of the position of the ring. The ring and the payload will have the same x-coordinate while the y-coordinate will differ by a value L_3 when the payload is in equilibrium. The condition on the constants K_{p_1} and K_{p_2} is $K_{p_1} > 0$ and $K_{p_2} > 0$. Therefore

$$\begin{aligned} \dot{V} = [K_{p_1} \int y_1 dt + K_{p_1}(x_R - x_D) + u_x](y_1 + y_3) \\ + [K_{p_2} \int y_2 dt + K_{p_2}(y_R - y_D - L_3) + u_y](y_2 + y_4) \end{aligned} \tag{4.19}$$

Let

$$u_x = -K_{p_1} \int y_1 dt - K_{p_1}(x_R - x_D) - K_{d_1}(y_1 + y_3) \tag{4.20}$$

$$\begin{aligned} u_y = -K_{p_2} \int y_2 dt - K_{p_2}(y_R - y_D - L_3) \\ -K_{d_2}(y_2 + y_4) \end{aligned} \tag{4.21}$$

Therefore,

$$\dot{V} = -K_{d_1}(y_1 + y_3)^2 - K_{d_2}(y_2 + y_4)^2 \tag{4.22}$$

From (4.22) it is observed that $\dot{V}$ is a negative semidefinite function (if $K_{d_1} \geq 0$ and $K_{d_2} \geq 0$) so that V is a positive semidefinite function. Hence for the SpiderCrane a function V exists that satisfies (3.1). Referring to Section 3, it can be said that the system becomes passive under the action of the controls u_x and u_y. The controls u_x and u_y are designed using output feedbacks rather than state feedbacks so that we do not require to check for the observability of any of the states. The positioning action is limited by the pendulum length L_3.

5. Simulation Results. The passivity based controller as derived in Section 4 is simulated assuming following values for the system parameters. The pendulum's angle with the vertical is denoted θ and it is desired that $\theta = 0$ along the trajectory as well as at the desired equilibrium. The values used are: payload mass $m = 1kg$, mass of ring $M = 0.5kg$, pendulum length $L_3 = 0.5m$. The desired payload location $(x_D, y_D) = (0.5, 1)m$. It is assumed that: initial position $(x_i, y_i) = (0.7, 0.7)m$, acceleration due to gravity $g = 9.81m/s^2$.
The simulation was carried out by selecting the following values for the controller constants: $K_{p_1} = 0.3$, $K_{p_2} = 0.08$, $K_{d1} = 2$ and $K_{d2} = 0.9$.
The selected values of the controller constants have been selected since they give fastest positioning of the load with minimum swing. The positioning time can be reduced if a larger swing of the payload is acceptable.
Results obtained for initial angle $\theta_i = 10^o$ are shown in Fig 3.

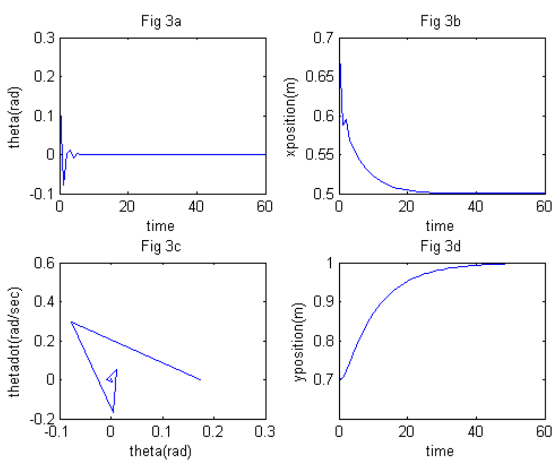


Fig 3: Plot of pendulum angle and ring position

From Fig 3a it is seen that the payload position goes from an initial position of 10^o to 0^o within a time of around 8 seconds. The x_R position moves from the initial value to the desired value of 0.5 in a manner as shown in Fig 3b. This transition is slightly oscillatory to compensate the pendulum's swing. The y_R position goes to the desired position of 1 in a smooth manner as seen in Fig 3d. Fig 3c shows the phase plot of pendulum position with respect to pendulum velocity i.e. θ w.r.t. $\dot{\theta}$. The trajectory converges to zero showing the stabilization of the payload.
Fig 4a shows the total force in the x direction which reduces to zero as the position x_R reaches the desired position. However the total force in the y direction (Fig 4b) does not reduce to zero as as the position y_R reaches it's desired value, since this force holds the payload against gravity. It is numerically equal to $(M + m)g$. The forces developed due to the actuators i.e. $\tilde{F}_x$ and $\tilde{F}_y$ reduce to zero as the desired position is reached. The kinetic energy (KE) (Fig 4d) has a nonzero value when the payload is in motion but reduces to zero when the payload stabilizes. The potential energy (PE) seen in Fig 4c stays at a finite value (corresponding to $(M + m)g - mgL_3 \cos\theta$).

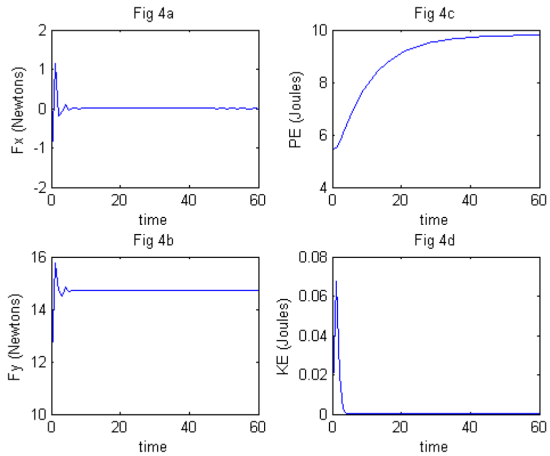


Fig 4: Plot of total forces and energy

The response of the controller to unmodeled parameters which acts as a disturbance was simulated. A disturbance occurs at time $t = 5$ seconds and at time $t = 39$ seconds. The payload position is seen in Fig 5a. Response in solid line indicates the payload position while that in dotted line indicates the disturbance signal. The response shows some oscillations at $t = 5$ seconds which are seen to be dying out completely by time $t = 11$ seconds. The second disturbance occurs at $t = 39$ seconds where we can see some oscillations. However these oscillations are highly damped and die out fast to give the desired payload position.

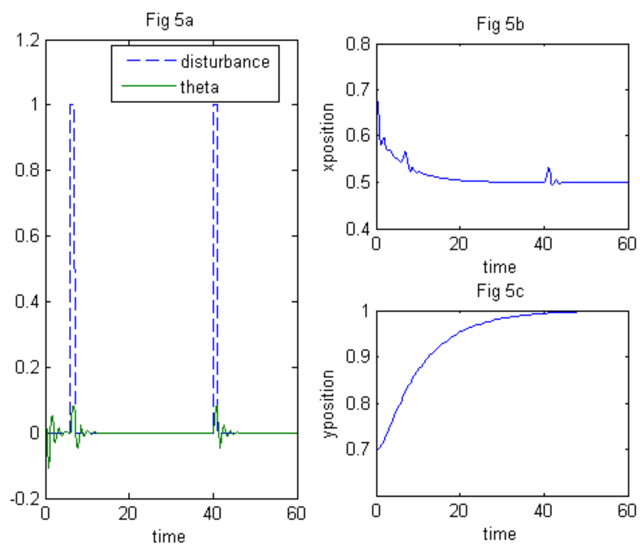


Fig 5:Payload and ring positions in presence of disturbance

The position x_R seen in Fig 5b shows oscillations when the disturbance occurs to balance the payload swing. The y_R position is unaffected by the disturbance as seen in Fig 5c.

6. Conclusions. In this paper it has been shown that the 2D SpiderCrane system is passive by establishing passivity of its pulley and payload dynamics and interconnecting them with a passive interconnection. A simple output feedback passivity based controller for the SpiderCrane system is presented which has been obtained without the formulation and solving of complex PDEs. The design was tested by means of simulation for a condition without any disturbance and also for an impulse like disturbance at payload. The simulations show precise payload positioning while keeping the swing minimum along the trajectory. The controller may be practically implemented using discrete time methods. The practical implementation needs to be checked for accuracy, noise immunity and robustness. Future work is aimed at incorporating the frictional forces between the cables and the pulleys.

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